

**M.Sc. DEGREE EXAMINATION, May 2015****(MATHEMATICS)****(SECOND YEAR)****210: COMPLEX ANALYSIS**

Time: Three hours

Maximum: 100 marks

**SECTION-A****(8×5=40)****Answer any EIGHT questions**

1. Prove that the limit function of uniformly convergent sequence of continuous functions is itself continuous.
2. State and prove Abels limit theorem.
3. State and prove Taylor's theorem.
4. State and prove the argument principle theorem.
5. If  $u_1$  and  $u_2$  are harmonic in a region  $\Omega$ , then prove that  $\int_{\gamma} u_1 du_2 - u_2 du_1 = 0$  for every cycle  $\gamma$  which is homologous to zero in  $\Omega$ .
6. State and prove the mean-value property for harmonic functions.
7. Prove that a necessary and sufficient condition for the absolute convergence of the product  $\prod_{n=1}^{\infty} (1 + a_n)$  is the convergence of the series  $\sum_{n=1}^{\infty} |a_n|$ .
8. Obtain Jensen's formula for an analytic function.
9. Prove that any two bases of the same module are connected by a unimodular transformation.
10. Prove that a nonconstant elliptic function has equally many poles as it has zeros.

**SECTION-B****(3×20=60)****Answer any THREE questions**

11. a) If all zeros of a polynomial  $P(z)$  lie in a half plane, then prove that all zeros of the derivative  $P'(z)$  lie in the same half plane.  
b) State and prove Cauchy's necessary and sufficient condition for uniform convergence of a sequence.
12. State and prove Cauchy's theorem for a rectangle.
13. State and prove Schwarz's theorem.
14. State and prove Arzela's theorem.
15. Obtain the differential equation satisfied by Weierstrass P-function.

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