Total No. of Pages: 2 **5277**

Register Number:
Name of the Candidate:

B.Sc. DEGREE EXAMINATION, May 2015

(MATHEMATICS)

(THIRD YEAR)

(PART – III)

710. VECTOR CALCULUS AND LINEAR ALGEBRA

Time: Three hours Maximum: 100 marks

Answer any FIVE questions **(5 × 20 = 100)**

1. a) If $\nabla\varphi = 2xyz^3\bar{i} + x^2z^3\bar{j} + 3x^2yz^2\bar{k}$, find φ(x, y, z).

   b) Show that ∇.(f×g)=g.(∇×f)–f.(∇×g).

2. a) Find the constants a, b, c so that the vector $\bar{f} = (x+2y+az)\bar{i} + (bx-3y-z)\bar{j} + (4x+cy+2z)\bar{k}$ is irrotational.

   b) If $\bar{f} = (2y+3)\bar{i} + xz\bar{j} + (yz-x)\bar{k}$, evaluate $\int_C \bar{f}.d\bar{r}$ along the path x=2t², y=t, z=t³ from t=o to t=1.

3. Verify Gauss divergence theorem for $\bar{f} = (x^2-yz)\bar{i} + (y^2-zx)\bar{j} + (z^2-xy)\bar{k}$ taken over the rectangular parallelepiped 0 ≤ x ≤ a, 0 ≤ y ≤b, 0 ≤ z ≤ c.

4. a) Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix}$ =(a–b)(b–c)(c–a)

   b) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

5. a) Define an idempotent matrix. Give an example.

   b) Using Cayley-Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

6. a) Prove that matrix $A=\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 1 & 0 & 1\end{pmatrix}$ satisfies the equation $A^3-3A^2+3A-2I=0$ and hence find A^4

   b) Verify that whether the matrix $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{pmatrix}$ is orthogonal.

7. a) Find the rank of the matrix $\begin{pmatrix}2 & -1 & 3 & 1\\ 1 & -2 & -1 & 4\\ 3 & 3 & 1 & 2\\ 6 & 0 & 3 & 7\end{pmatrix}$

   b) Prove that the following equations are consistent and hence solve them. $X+2y-z=1$; $3x+8y+2z=28$; $4x+9y-z=14$.

8. a) Show that the intersection of two subspaces of a vector space is a subspace and the union of two subspaces of a vector space need not be a subspace.

   b) Let A and B be two subspaces of a vector space V. Prove that $A\cap B=\{0\}$ if and only if every vector $\upsilon \in A+B$ can be uniquely expressed in the form $\upsilon = a+b$ where $a\in A$ and $b\in B$.

9. a) Let V be a vector space over a field F. Let A and B be subspaces of V. Prove that $\frac{A+B}{A}\cong\frac{B}{A\cap B}$

   b) Prove that the vectors (1,1,0), (0,1,1),(1,0,1) are linearly independent in $V_3(R)$.

10. a) Show that any two bases of a finite dimensional vector space V have the same number of elements.

    b) Prove that $\dim\frac{v}{w}=\dim v-\dim w$.
